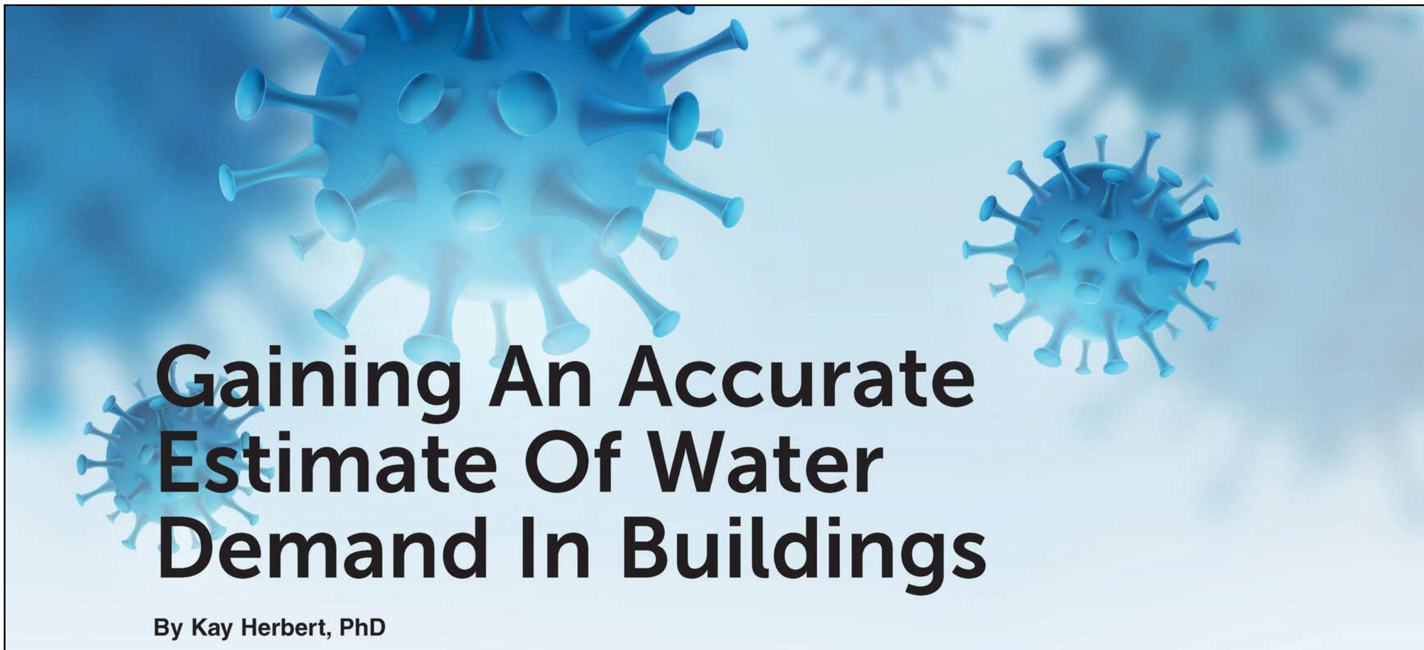

Gaining An Accurate Estimate Of Water Demand In Buildings

By Kay Herbert, PhD

Estimated peak water demand is customarily used to size a building's piping system. But with Covid-19 and other pathogens, a higher demand for sensor-activated and high efficiency fixtures has increased the need for more accurate demand calculations given a large variety of water dispensing fixtures.

A new approach to adapt the known method of convolution on discrete distributions is introduced for accurate water demand estimation. The new approach finds the cumulative distribution function (CDF) of water demand for an arbitrary number of different fixture groups with different probabilities of activation and different flow rates. The method is accurate for smaller buildings and individual restrooms with few fixtures, as well as for large buildings with high fixture counts. Using this approach, a design curve to look up the estimated peak for water demand will no longer be needed.

Peak Water Demand at a Glance

Accurate estimates of peak water demand have long been an essential aspect of designing a building's plumbing system. While accurate sizing of the plumbing system requires it to be large enough to handle peak demand, it is also known that stagnant water promotes growth of waterborne diseases like Legionella, emphasizing the need to optimize a plumbing system. Making accurate demand calculations even more critical has been the exponential growth of "green" buildings with reduced water demand. In fact, a recent study by Rhoads, Pruden and Edwards[1] found that increased water age due to reduced consumption is relevant to the stability of water quality.

In 1940, Roy Hunter[2] published his ground-breaking paper on estimating loads in plumbing systems. Hunter used a clever method of collapsing the demand curves of different plumbing fixture types onto one approximate curve in order to account for the differences in both probabilities and flow rates.

While Hunter's approximation received worldwide acceptance, it is not fully accurate, especially considering modern plumbing systems. While many improvements have been made to Hunter's approximation over the past 80 years, none have been deemed universally accurate.

Yet, by using the method of convolution, it is now possible to get perfectly accurate results via direct numerical calculations for both small and large plumbing systems.

Hunter's Curve

Hunter started with a few premises: every plumbing fixture essentially has an expected on-time where the fixture draws water at a flow rate q for a duration time t. Additionally, every fixture has a minimum expected time between uses of duration T, therefore the fixture has an expected off time of T-t between uses. The minimum time T between successive activations and the active time t of the same fixture can be a function of the number of people in a building, the behaviour of people using it, or properties of the type of fixture.

Taking a random sample, we would find a fixture in a building dispensing water with a probability $p = t/T$. If a multiple number n of fixtures with the same probability and flow rate are installed in a building, then the probability P that x fixtures (x=0, 1, 2, 3…) are operating at the same time is given by the binomial probability density function (PDF):

Equation 1

$$P[x;n,p] = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{where } \binom{n}{x} = \frac{n!}{(n-x)!x!}, x = 0, 1, 2 \ldots n$$

The cumulative distribution function or

$$C[k] = \sum_{x=0}^{x=k} P[x;n,p]$$

then provides the probability that k or less fixtures are operating at the same time. Note that $C[0] = (1-p)^n$ is the probability that all fixtures are off, and $C[n]=1$ is the probability any or all fixtures are on. Hunter proposed to use the 99th percentile ($C[k] \geq .99$) of the cumulative density function of the distribution to determine the peak number of fixtures k in use.

The expected peak flow rate is simply k (99th percentile probability of fixtures n) times q (the fixture flow rate when it is on).

Buildings typically have multiple fixture types with

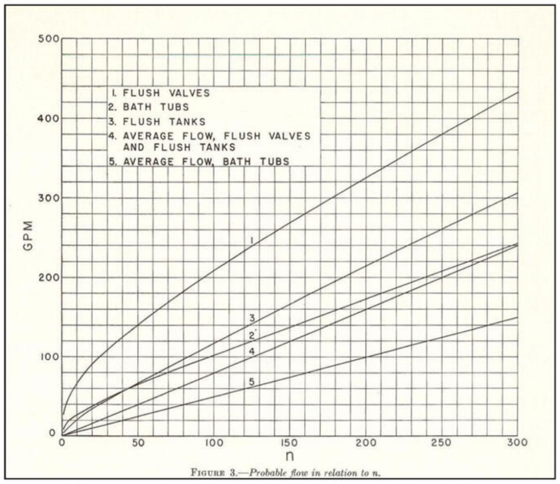

Figure 1 Hunter's original 1940 calculation of 99th percentile curves

different probabilities and flow rates. While it is easy to obtain the CDF for a single fixture type, the probability functions cannot just be added to obtain the PDF or CDF of a plumbing system with different types of fixtures. Hunter solved this dilemma by assigning arbitrary "fixture units" to each fixture type, in a way that the same number of fixture units for one type of fixture results approximately in the same 99th percentile flow rate for the same number of fixture units of other fixtures.

Recent Advances

Estimating peak demand can be more than 20 percent wrong using Hunter's method with modern fixtures. That is why the need for improved accuracy prompted Wistort[3] to propose the use of normal distributions approximating binomial distributions. While this method eliminates creating arbitrary fixture units and somewhat improves accuracy, it also has diminishing accuracy for small numbers of fixtures with small probabilities.

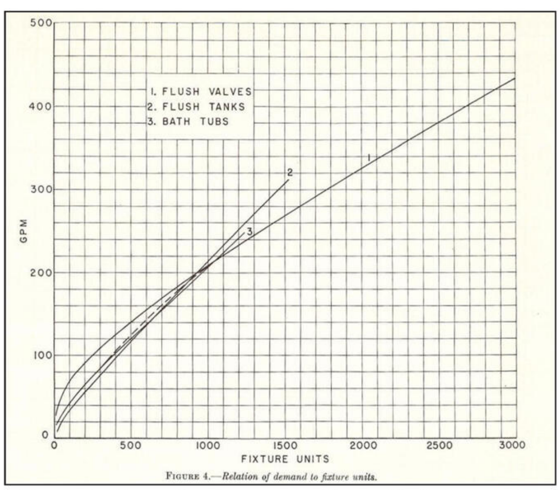

Figure 2 Hunter's clever rescaling with fixture units

Omaghomi and Buchberger[4] discuss Wistort's method in more detail and discuss alternate methods such as exact enumeration and a Monte Carlo method.

Exact enumeration requires 2^n (where n is the total number of fixtures) calculations to find water demand and therefore becomes computationally too expensive for larger n. The Monte Carlo method requires many computations to produce more accurate results and it is difficult to ascertain the required accuracy.

Accurate Method by Convolution

It has been shown that it is possible to calculate the exact distribution as the sum of two discrete random distributions by taking the convolution of the two:

Equation 2

$$P[j; n = n1 + n2] = \sum_{i=0}^{j} P1[i; n1] \cdot P2[j - i; n2]$$

Calculating the convolution is much more efficient than enumeration, as it requires only the order of n^2 multiplications. For k different distributions Equation 2 can be applied successively k-1 times to obtain the combined probability distribution as the sum of all k distributions.

To demonstrate this, the test case of n=15 fixtures and k=6 fixture groups as shown in Table 1 (taken from Omaghomi) as a test case here:

Table 1

k	nk	pk	qk (gpm)
1	2	.125	4.0
2	1	.140	3.0
3	1	.100	2.5
4	6	.200	1.5
5	1	.125	2.0
6	4	.067	2.4

Using Equation 1 to obtain the distribution for each fixture group and then repeatedly applying Equation 2, the probabilities of all fixture combinations can be found, and the resulting CDF can be obtained. Table 2 and Figure 3 shows the resulting probability distribution (rounded to 6 decimals) and the resulting cumulative distribution function (rounded to 4 decimals).

Table 2

n	p	CDF
0	0.102999	0.103
1	0.256438	0.3595
2	0.293701	0.653
3	0.205137	0.8585
4	0.097657	0.956
5	0.033544	0.9895
6	0.008583	0.998
7	0.001665	0.9995
8	0.000247	1
9	0.000028	1
10	2.5E-06	1
11	0	1
12	0	1
13	0	1
14	0	1
15	0	1

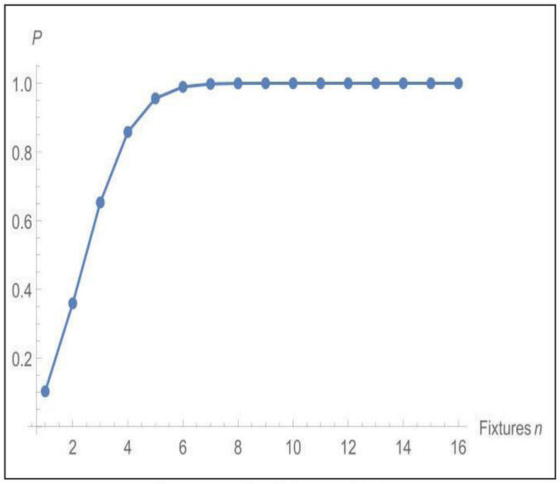

Figure 3: CDF from Table 2

While the above method obtains the probability for an outcome of *n* (*n*=0,1,2…15) fixtures operating simultaneously, it is not useful to obtain the probability of outcome based in flow rate. Different fixtures not only can have different probabilities of operating, but they also can have different flow rates. Just multiplying the number of fixtures by a peak or average flow rate will not guarantee the correct distribution.

In order to get an accurate distribution for flow rates as the outcome, all distributions must have the same discrete flow rate scale prior to obtaining the distribution sum via the convolution (Equation 2).

An efficient method is to choose a consistent flow rate interval *dq* where *dq* is chosen so that the flow rate *qi* for every fixture group *i* divided by *dq* results in an integer mi i.e. $qi/dq = m_i$. The probability density function for a unique fixture group i with flow rate q_i and probability p_i is then given by:

Equation 3

$$P[l\,dq; n_i, p_i] = \binom{n_i}{x} {p_i}^x (1-p_i)^{n_i-x} \; for\; l = x\, m_i, x = 0 \ldots n_i \; and$$

$$P[l\,dq; n_i, p_i] = 0 \; for\; l \neq x\, m_i, \quad l = 0,1 \ldots n_i\, m_i$$

For the example of Table 1, with fixture group 6 (*k*=6), the flowrate distribution is given in Figure 4 with *dq*=.1 gpm. For a single fixture group, the probability density function is very sparse for small dq. Multiple convolutions with all the other fixture groups of different flow rates makes the distribution more populated. Figure 5 shows the resulting probability density function (PDF) for all fixture groups from Table 1.

The cumulative density function (CDF) can be obtained in the usual manner summing the PDF as shown in Figure 6. The 99th percentile flowrate q_{99} (or any other percentile) can then be found. In this example q_{99} = 12.7 gpm.

For many fixtures, the same method gives accurate results as well. Using the same parameters from Table 1 except multiplying all fixture counts by 100 (for a total of 1,500 fixtures), the probabilities (Figure 7) and the CDF (Figure 8) are obtained.

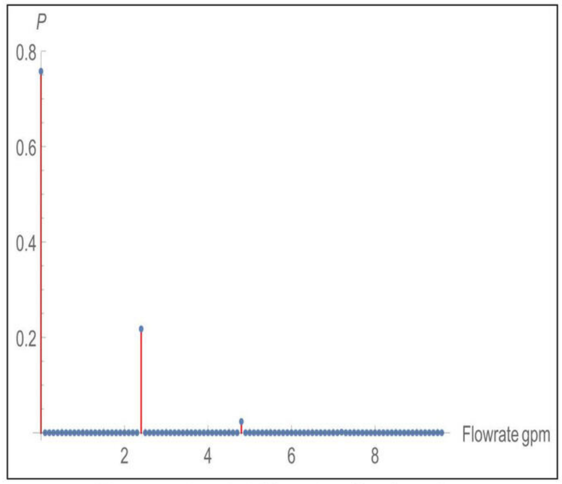

Figure 4: Probability density function with flowrate as outcome

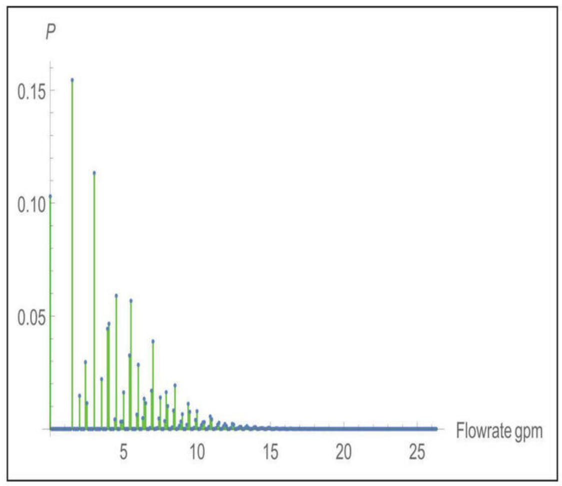

Figure 5: example probability density function by convolution

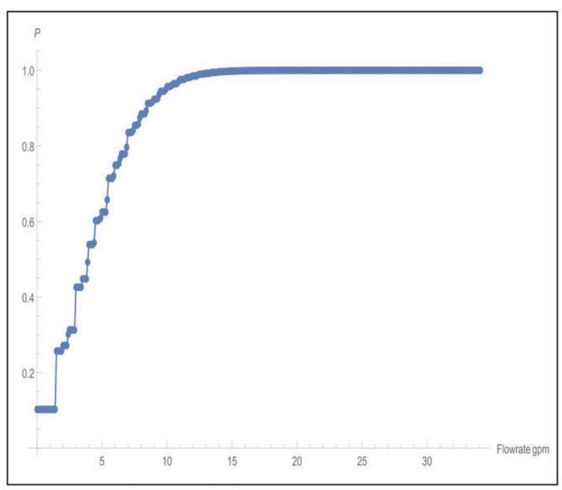

Figure 6 CDF for above example

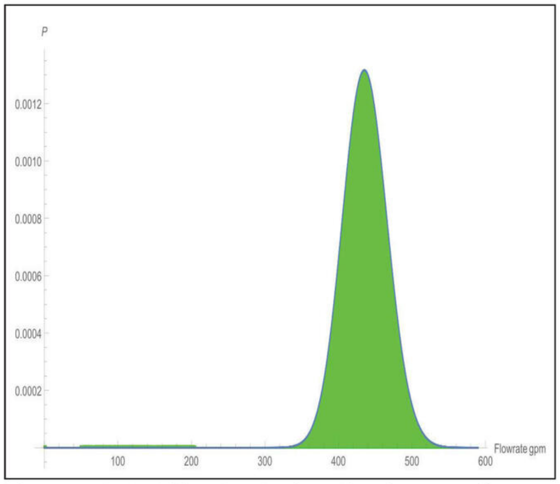

Figure 7 Probability density function for 1,500 fixtures

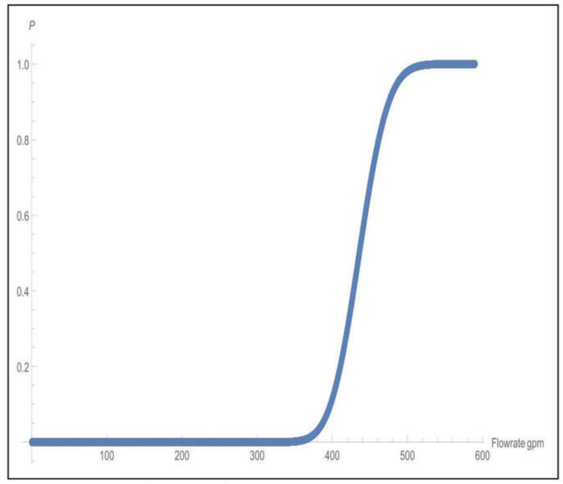

Figure 8 CDF for 1,500 fixtures

It was found that in this case q_{99} = 508.4 gpm.

All calculations were performed on a Lenovo X1 Extreme laptop and took only seconds to complete when programmed and run in Wolfram Mathematica. It should be noted that memory requirements and computation time can be reduced by truncating the ends of large PDF arrays where probabilities are extremely low, if needed.

This method allows us to analyse demand anywhere from single restrooms and water dispensing zones to commercial buildings, making approximations like Hunter's curve and Wistort's formula obsolete.

Fixture usage probabilities and flow rates can be a function of time, temperature, pressure, type of usage, and human behaviour. A better understanding of these mechanisms drives the future design of modern restrooms and will maximize sustainability and health. ●

Kay Herbert is Sloan's chief scientist. He works to aid in the research and development of new technologies and products to help bring innovative solutions to the restroom.

References

1. William J. Rhoads, Amy Pruden and Marc A. Edwards Survey of green building water systems reveals elevated water age and water quality concerns; Environmental Science, Water Research & Technology, 2016,2,164

2. Hunter, R.B. Methods of Estimating Loads on Plumbing Systems; Report BMS65; National Bureau of Standards: Washington, DC, USA, 1940

3. R.A. Wistort, "A new look at determining water demands in building: ASPE direct analytic method," in Technical Proc. American Soc of Plumbing Engineers Convention, Kansas City, MO, ASPE, 1994, pp 17-34

4. T. Omaghomi, S. Buchberger Estimating Water Demands in Buildings, 16th Conference on Water Distribution Systems Analysis, WDSA 2014 Procedia Engineering 89 (2014)1013-1022